17/12/20

B.Sc. Part - I

Page No.:
Date: / /

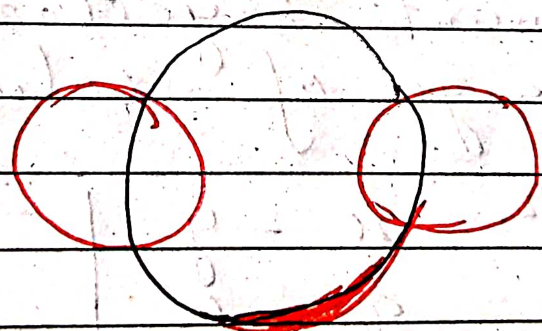
To find the General equation of the circle cutting two given circles Orthogonally.

Let the two circles have for their equations $x^2+y^2+2gx+2fy+c=0$ and

$$x^2+y^2+2g'x+2f'y+c'=0$$

Let the equation to a circle cutting them orthogonally be

$$x^2+y^2+2ax+2by+K=0 \quad -(1)$$



Then the conditions for Orthogonality are

$$2ag+2bf-c-K=0 \quad -(2)$$

& $$2ag'+2bf'-c'-K=0 \quad -(3)$$

Eliminating a & b from (1) (2) & (3) we get

$$\begin{vmatrix} x^2+y^2+K & x & y \\ -c-K & g & f \\ -c'-K & g' & f' \end{vmatrix} = 0$$

Or, $$\begin{vmatrix} x^2+y^2 & x & y \\ -c & g & f \\ -c' & g' & f' \end{vmatrix} + \begin{vmatrix} K & x & y \\ -K & g & f \\ -K & g' & f' \end{vmatrix} = 0$$

Now multiply the first determinant by -1 in the first row and the first column and doing the same of the second determinant together with interchange of column we get

$$\begin{vmatrix} x^2+y^2 & -x & -y \\ c & g & f \\ c' & g' & f' \end{vmatrix} + K \begin{vmatrix} -x & -y & 1 \\ g & f & 1 \\ g' & f' & 1 \end{vmatrix} = 0$$

This is the general equation for all such circles, K being an arbitary constant.